2501/103 2508/103
2502/103 2509/103
2503/103
ENGINEERING MATHEMATICS I
June/July 2018
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN MECHANICAL ENGINEERING
(PRODUCTION OPTION)
(PLANT OPTION)
DIPLOMA IN AUTOMOTIVE ENGINEERING
DIPLOMA IN WELDING AND FABRICATION
DIPLOMA IN CONSTRUCTION PLANT ENGINEERING

MODULE I

ENGINEERING MATHEMATICS I

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Mathematical tables/ Non-programmable scientific calculator;
*This paper consists of **EIGHT** questions.*
*Answer any **FIVE** questions in the answer booklet provided.*
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Simplify the expressions:

(i) $$\frac{(1+x)^{\frac{1}{2}}-(1+x)^{-\frac{1}{2}}}{(1+x)^{\frac{3}{2}}}$$

(ii) $$\frac{\log 625-\frac{1}{2}\log 25}{\log 125+\frac{1}{2}\log 25}$$

(7 marks)

(b) Solve the equations:

(i) $2^{2x+1}=8^{\frac{4x}{3}}$

(ii) $\log_x 4+2\log_2 x^2=6$

(13 marks)

2. (a) Find the ratio of the term in x^5 to the term in x^3 in the binomial expansion of $(2x+5)^{10}$, and determine its value when $x=\frac{1}{3}$, correct to four decimal places.

(8 marks)

(b) Determine the first four terms in the binomial expansion of $(1+2x)^{\frac{1}{2}}$, and state the values of x for which the expansion is valid.

(4 marks)

(c) (i) Use the binomial theorem to show that, if x is very small, then

$$\sqrt{\left(\frac{1-\frac{1}{2}x}{1+\frac{1}{2}x}\right)}=1-\frac{1}{2}x+\frac{1}{8}x^2-\frac{1}{16}x^3.$$

(ii) By setting $x=\frac{1}{2}$ in the result in (i), determine the appropriate value of $\sqrt{0.6}$, correct to four decimal places.

(8 marks)

3. (a) Solve the equations:

(i) $\frac{3}{x-2}+\frac{2}{x-3}=2$

(ii) $3(2^{2x})-7(2^x)+2=0$

(12 marks)

(b) Three forces F_1, F_2 and F_3 in newtons necessary to keep a certain mechanical system in equilibrium satisfy the simultaneous equation:

$$2F_1 - F_2 + F_3 = 3$$
$$-F_1 + 2F_2 + 2F_3 = -3$$
$$3F_1 - 2F_2 + F_3 = 2$$

Use the method of elimination to determine the values of the forces.

(8 marks)

4. (a) Prove the identities:

(i) $\dfrac{1+\cos\theta}{\sin\theta} = \dfrac{\sin\theta}{1-\cos\theta}$

(ii) $\cos\theta + \cos 2\theta + \cos 3\theta = \cos 2\theta\,(2\cos\theta + 1)$

(6 marks)

(b) (i) Given $\sin(\theta+\alpha) = 2\cos(\theta-\alpha)$, show that $\tan\theta = \dfrac{2+\tan\alpha}{1+2\tan\alpha}$.

(ii) Hence solve the equation

$\sin(\theta + \frac{\pi}{4}) = 2\cos(\theta - \frac{\pi}{4})$, for values of θ between 0° and 360° inclusive.

(6 marks)

(c) Solve the equation:

$3\cos 2\theta + \sin\theta + 2 = 0$, for values of θ between 0° and 360° inclusive.

(8 marks)

5. (a) Determine the values of M and N such that $5\cosh x - 3\sinh x = Me^x + Ne^{-x}$.

(4 marks)

(b) (i) Derive the identity:

$\cosh(x+y) = \cosh x\cosh y + \sinh x \sinh y$.

(ii) Use Osborne's rule to derive the identity for $\coth^2 x$ from the trigonometric identity: $1 + \cot^2 x = \operatorname{cosec}^2 x$.

$3(\cosh^2 x + \sinh^2 x) - \sinh x - 7 = 0$

(8 marks)

(c) Solve the equation:

$3\cosh^2 x + 3\sinh^2 x) - \sinh x - 7 = 0$

$3\cosh 2x - \sinh x - 7 = 0$

$3 - \sinh x - 7 = 0$

(8 marks)

$\sinh x = -4$

$\frac{e^x - e^{-x}}{2} = 4$

$e^{2x} - 1 = 8e^x$

$e^{2x} - 8e^x - 1$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

6. (a) Given the function $f(x)=\frac{3-x}{x+4}$, determine:

(i) $f^{-1}(0)$

(ii) $f^{-1}\left(-\frac{1}{2}\right)$

(7 marks)

(b) (i) Show that $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$.

(ii) Hence determine the value of $\tan^{-1}(1)+\tan^{-1}(\sqrt{3})$.

(8 marks)

(c) By expressing $\sinh^{-1}x$ in logarithmic form, determine the value of $\sinh^{-1}(6)$.

(5 marks)

7. (a) Given the complex numbers $Z_1=-1+2j$, $Z_2=1+j$ and $Z_3=\frac{1}{Z_1}+\frac{1}{Z_2}$, express Z_3 in polar form.

(8 marks)

(b) Given that $Z=j$ is one root of the equation $Z^3+3Z^2+Z+3=0$, determine the other roots.

(5 marks)

(c) Solve the equation:

$Z^4+1+j\sqrt{3}=0$, giving the answers in polar form.

(7 marks)

8. (a) The sum of the first three terms of an arithmetic progression is 3, and the difference between the seventh term and the fourth term is -6. Determine the:

(i) first term and common difference

(ii) sum of the first thirty terms.

(6 marks)

(b) The third term of a geometric progression is eight times the sixth term, and the sum of the second and fifth terms is $\frac{9}{16}$. Determine the:

(i) first term and common ratio

(ii) sum of the first ten terms.

(7 marks)

(c) Express the equation of the parabola $y^2=4-4x$ in polar form.

(7 marks)

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